Neutrinoless Double Beta Decay and Future Neutrino Oscillation Precision Experiments

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Abstract

We discuss to what extent future precision measurements of neutrino mixing observables will influence the information we can draw from a measurement of (or an improved limit on) neutrinoless double beta decay. Whereas the Δm^2 corresponding to solar and atmospheric neutrino oscillations are expected to be known with good precision, the parameter θ_{12} will govern large part of the uncertainty. We focus in particular on the possibility of distinguishing the neutrino mass hierarchies and on setting a limit on the neutrino mass. We give the largest allowed values of the neutrino masses which allow to distinguish the normal from the inverted hierarchy. All aspects are discussed as a function of the uncertainty stemming from the involved nuclear matrix elements. The implications of a vanishing, or extremely small, effective mass are also investigated. By giving a large list of possible neutrino mass matrices and their predictions for the observables, we finally explore how a measurement of (or an improved limit on) neutrinoless double beta decay can help to identify the neutrino mass matrix if more precise values of the relevant parameters are known.

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1 Introduction

The last few years have seen tremendous progress in our understanding of the neutrino sector. The solar neutrino deficit is now known to be certainly due to neutrino flavor oscillations [1, 2, 3, 4, 5, 6] with the best–fit oscillation parameters given as $\Delta m_\odot^2 \equiv \Delta m_{21}^2 = 8.0 \times 10^{-5}$ eV^2 and $\sin^2 \theta_\odot \equiv \sin^2 \theta_{12} = 0.31$ [5]. The atmospheric neutrino deficit can also be ascribed to flavor oscillations with a good degree of confidence [7, 8] with the best–fit oscillation parameters as $\Delta m_{atm}^2 \equiv |\Delta m_{31}^2| = 2.1 \times 10^{-3}$ eV^2 and $\sin^2 2\theta_{atm} \equiv \sin^2 2\theta_{23} = 1$ [7]. The upper limit on the third mixing angle θ_{13} is mainly determined by the reactor neutrino data [9], which when combined with the information obtained from the solar and atmospheric neutrino experiments gives a bound of $\sin^2 \theta_{13} < 0.044$ at 3σ [10, 11].

Despite all these impressive achievements, there still remains a lot to be learned. The arguably most fundamental question, namely whether neutrinos are Dirac or Majorana particles, remains still to be answered. If neutrinos are Majorana particles, we have nine physical parameters describing the 3×3 light neutrino mass matrix and determining them is one of the ultimate goals of neutrino physics. With the two Δm^2 and two mixing angles – θ_{12} and θ_{23} — known, there still remain five parameters to be tracked down. To be more precise, we still lack knowledge about the following points:

- Is there CP violation in the lepton sector in analogy with that in the quark sector?

- Is the atmospheric neutrino mixing angle θ_{23} exactly maximal? If not, does it lie above or below $\pi/4$?

- Is the third mixing angle θ_{13} exactly zero?

- How small is the absolute neutrino mass scale?

- What is the ordering of the neutrino masses, i.e., what is the neutrino mass hierarchy?

Answers to all or some of these question will certainly lead to better understanding of the underlying theory which gives rise to neutrino masses[1].

Neutrinoless double beta decay ($0\nu\beta\beta$) is expected to be of crucial importance in answering some of the questions raised above. At the same time, answers to some the questions obtained from elsewhere will help in interpreting a positive or negative signal in the next generation $0\nu\beta\beta$ experiments. Goal of these experiments is the observation of the process

$$(A, Z) \rightarrow (A, Z - 2) + 2\, e^- \ .$$

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[1]A rarely mentioned point is that for Majorana neutrinos one can, at least in principle, fully determine the complete mass matrix because it is symmetric. If the neutrinos are Dirac particles like quarks and charged leptons, their mass matrix is in general not symmetric and therefore parametrized by fifteen free parameters. However, since we only see left–handed weak currents, the "right–handed" parts of the neutrino parameters are out of reach and hence not observable.

The effective mass which will be extracted or bounded in a $0\nu\beta\beta$ experiment is given by the following coherent sum:

$$\langle m \rangle = \left| \sum_i m_i U_{ei}^2 \right| , \tag{1}$$

where m_i is the mass of the ith neutrino mass state, the sum is over all the light neutrino mass states and U_{ei} are the matrix elements of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix [13]. We parametrize it as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}) , \tag{2}$$

where we have used the usual notations $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, δ is the Dirac CP–violation phase, α and β are the two Majorana CP–violation phases [14]. Thus we can define $\langle m \rangle$ in terms of the oscillation parameters, the Majorana phases and the neutrino mass scale. This means that $\langle m \rangle$ depends on 7 out of 9 parameters contained in the neutrino mass matrix. In particular, the effective mass is a function of all unknowns of neutrino physics except for the Dirac phase and θ_{23}. This means that a measurement of – or a stringent limit on – $0\nu\beta\beta$ will give us some information on the unknown parameters. Combined with complementary information from other independent types of experiments, this would reinforce our understanding of the neutrino sector.

There is a large list of available analyzes focusing on the connection of the known and unknown neutrino parameters with the effective mass [15, 16, 17, 18, 19, 20], for a recent review of the theoretical situation see [21]. In this work we want to focus on the mass ordering of the neutrinos, on the neutrino mass scale and on the implications of a very small or vanishing effective mass. The effective mass to be extracted from neutrinoless double beta decay depends crucially on the neutrino mass spectrum. Of special interest are the following three extreme cases:

$$\text{normal hierarchy (NH):} \quad |m_3| \simeq \sqrt{\Delta m_{\mathrm{A}}^2} \gg |m_2| \simeq \sqrt{\Delta m_{\odot}^2} \gg |m_1| , \tag{3}$$

$$\text{inverted hierarchy (IH):} \quad |m_2| \simeq |m_1| \simeq \sqrt{\Delta m_{\mathrm{A}}^2} \gg |m_3| , \tag{4}$$

$$\text{quasi–degeneracy (QD):} \quad m_0 \equiv |m_3| \simeq |m_2| \simeq |m_1| \gg \sqrt{\Delta m_{\mathrm{A}}^2} . \tag{5}$$

The order of magnitude of the effective mass in those spectra is $\sqrt{\Delta m_{\odot}^2}$, $\sqrt{\Delta m_{\mathrm{A}}^2}$ and m_0, respectively.

The best current limit on the effective mass is given by measurements of ^{76}Ge established by the Heidelberg–Moscow collaboration [22]

$$\langle m \rangle \leq 0.35 \, z \, \mathrm{eV} , \tag{6}$$

where $z = \mathcal{O}(1)$ indicates that there is an uncertainty stemming from the nuclear physics involved in calculating the decay width of $0\nu\beta\beta$. Similar results were obtained by the IGEX collaboration [23]. Several new experiments are currently running, under construction or in the planing phase. The NEMO3 [24] and CUORICINO [25] experiments are already taking data and reach sensitivities near the current limits. The next generation of experiments, with projects such as CUORE [26], MAJORANA [27], GERDA [28], EXO [29], MOON [30], COBRA [31], XMASS, DCBA [32], CANDLES [33], CAMEO [34], (for a review see [35]) will probe values of $\langle m \rangle$ one order of magnitude below the limit from Eq. (6). Thus we expect $\langle m \rangle$ to be probed down to $\simeq \sqrt{\Delta m_{\rm A}^2} \simeq 0.04$ eV and it would be pertinent to ask if such a measurement could help us learn about the neutrino hierarchy[2].

Maximal mixing in the solar neutrino sector is now disfavored at almost 6σ and this non–maximality of θ_{12} allows for the possibility to distinguish the NH from the IH case. This however depends on the value of the smallest neutrino mass state. We investigate this issue and give the value of the smallest neutrino mass state which allows to distinguish NH from IH. Another aspect, concerning the QD spectrum, is the size of the absolute neutrino mass scale. We give and analyze a simple formula for a limit on m_0 in terms of θ_{12}, θ_{13} and the limit (or value) of $\langle m \rangle$. All these issues depend crucially on the uncertainties involved. In addition to the experimental errors, we would have to contend with the theoretical uncertainties coming from unknown nuclear matrix elements and uncertainties in the values of the oscillation parameters. As far the oscillation parameters are concerned, the uncertainties are expected to be reduced sharply by the next generation experiments. This will greatly reduce the expected range of $\langle m \rangle$ for a given mass spectrum. We give a detailed review of the improvements expected in each of the oscillation parameters and identify the parameters which would still lead to maximum uncertainty in the predicted value of $\langle m \rangle$ for a given mass spectrum. We take the uncertainties coming from our lack of knowledge of the nuclear matrix elements and explore the chances of determining the neutrino mass hierarchy from a positive $0\nu\beta\beta$ signal in the future. Furthermore, we analyze the consequences of a very small or even vanishing $\langle m \rangle$ if neutrinos are indeed Majorana particles. Such a small $\langle m \rangle$ will influence the allowed range of values of the mixing parameters θ_{12}, θ_{13}, the Majorana phases and the smallest neutrino mass. We compare these implied range of parameter values with their current limits. Finally, we perform a thorough scan of the phenomenologically viable neutrino mass models and give a list of possible neutrino mass matrices out of which the true one might be identified when future precision measurements of neutrino oscillation observables are combined with a measurement of or limit on $\langle m \rangle$.

We begin by giving in Section 2 a detailed report of the existing range of values for the oscillation parameters. We list down the expected improvements in each of the parameters. In Section 3 we probe the chances of distinguishing the IH from the NH from a future positive signal for neutrinoless double beta decay. We take into account the current and future

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[2]Of course those experiments aim also to put the controversial [36] evidence of part of the Heidelberg–Moscow collaboration to the test.

expected uncertainties on the oscillation parameters and include the role of the uncertainty in the nuclear matrix elements. In Section 4 we consider QD as the true mass spectrum of the neutrinos and study what limits one could set on the absolute/common neutrino mass scale m_0 from a measured value of $\langle m \rangle$ and the current and future uncertainties on the oscillation parameters. Section 5 probes the situation one would have if we do not see any signal in the next set of $0\nu\beta\beta$ experiments and yet believe that neutrinos are Majorana particles. In Section 6 we then list viable neutrino mass matrices and discuss how the number can drastically be reduced when measurements of $\langle m \rangle$ and the other oscillation parameters have been performed. We finally conclude in Section 7.

2   Past, Present and Future data

In this Section we shall present the status of the current global neutrino data and the prospects for its future improvement. First we give the current situation.

At the 3σ level, our current knowledge of the solar parameters within 3σ is limited to [5, 10]

$$7.0 \times 10^{-5} \text{ eV}^2 < \Delta m_\odot^2 < 9.3 \times 10^{-5} \text{ eV}^2 \ , \tag{7}$$

$$0.24 < \sin^2 \theta_{12} < 0.41 \ . \tag{8}$$

The atmospheric mass squared difference and mixing angle at 3σ are known within [7, 11]

$$1.3 \times 10^{-3} \text{ eV}^2 < \Delta m_{\text{A}}^2 < 4.2 \times 10^{-3} \text{ eV}^2 \ , \tag{9}$$

$$0.33 < \sin^2 \theta_{23} < 0.66 \qquad \ , \tag{10}$$

while the mixing angle θ_{13} at 3σ is restricted to lie below the value [9, 10, 11, 37]

$$\sin^2 \theta_{13} < 0.044 \ . \tag{11}$$

A very useful parameter related to the mass hierarchy of the neutrinos is the ratio of the solar and atmospheric mass squared differences,

$$R \equiv \frac{\Delta m_\odot^2}{\Delta m_{\text{A}}^2} \ , \text{ with } 0.017 < R < 0.072 \ \text{ at } 3\sigma \ . \tag{12}$$

Regarding the absolute value of the neutrino masses, several approaches exist. The most model–independent one is certainly the direct search for kinematical effects in the energy spectra of beta–decays. The Mainz [38] and Troitsk [39] experiments gave upper limits on the electron neutrino mass of 2.3 eV at 95 % C.L. Cosmological observations imply typically more stringent limits, which however considerably depend on the data set and the priors used in the analysis. Combining the cosmic microwave radiation measurements by the WMAP satellite [40] with data on the large scale structure of the Universe and other data sets, gives limits on the sum of neutrino masses between 0.4 and 2 eV, see [12] for a recent update of the situation.

Finally, there is currently no information on any of the three possible CP phases, which take on values between zero and 2π.

Let us now discuss the current uncertainties in the values of the parameters and their expected improvement. We can see from Eqs. (7) and (9) that we still have 53% (14%) uncertainty[3] on the value of Δm_{31}^2 (Δm_{21}^2) at 3σ. These ranges of allowed values are expected to reduce remarkably in the future: the uncertainty in Δm_{21}^2 is expected to reduce to 7% at the 3σ level from future measurement of reactor antineutrinos in KamLAND [37, 41] and the value of Δm_{31}^2 is expected to be known with $\sim 5\%$ at the 3σ level from the next generation accelerator experiments involving superbeams in the next ten years [42].

The uncertainty in the knowledge of $\sin^2\theta_{23}$ is at present 33% at 3σ. This is expected to reduce to $\sim 20\%$ in the next ten years after the results from the next generation superbeam experiments become available [42]. It is clear that the mixing angle θ_{23} plays no role for the study of neutrinoless double beta decay. Nevertheless, we shall see later on in Section 6 that in order to identify the neutrino mass matrix it can be rather important to know "how maximal" θ_{23} actually is. Of particular importance in this respect is the question whether the atmospheric neutrino mixing is exactly maximal because this would point to the presence of an underlying symmetry. This deviation will be known to order 10% after the next generation long–baseline experiments [43]. Comparative constraints are expected from future SK atmospheric neutrino data with statistics 50 times the current SK statistics [44]. Another way to state this is that we can identify if $\theta_{23} - \pi/4$ is of order $\sqrt{R}$ or smaller with next generation long–baseline experiments. A deviation from $\pi/4$ even smaller (i.e., order R) can be achieved by dedicated next generation experiments [43, 44, 45] (see also [46]).

The effective mass in neutrinoless double beta is crucially dependent on the values of the mixing parameters θ_{12} and θ_{13}. The current uncertainty on the values of $\sin^2\theta_{12}$ is 29% at the 3σ level while $\sin^2\theta_{13}$ is completely unknown. The uncertainty on the value of $\sin^2\theta_{12}$ and the limit on $\sin^2\theta_{13}$ is however expected to reduce in the future. While the conventional beam experiments, MINOS, ICARUS and OPERA are expected to provide moderate improvement on $\sin^2\theta_{13}$ [42], the Double–Chooz reactor experiment in France is expected to reduce the upper limit to $\sin^2\theta_{13} \lesssim 0.008$ at the 3σ level [47]. This upper limit could be improved further to $\sin^2\theta_{13} \lesssim 0.0025$ by the combination of the next generation beam experiments T2K [48] and NOνA [49], as well as by the second generation reactor experiments [50]. It should be borne in mind that any one of these experiments could even measure a non–zero θ_{13}, if the true value of θ_{13} happens to fall within their range of sensitivity.

On the other hand, the improvement expected for $\sin^2\theta_{12}$ from the currently running experiments is small [41, 51]. The results from the third and the final Helium phase of the ongoing SNO experiment are expected to reduce the uncertainty on $\sin^2\theta_{12}$ to not more than $\sim 21\%$ [37, 52] and the KamLAND experiment is not expected to make any

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[3]We define uncertainty of a quantity as the difference of the maximally and minimally allowed value divided by its sum and then multiplied by 100.

significant improvement on it [41, 51, 53]. Doping the Super–Kamiokande (SK) detector with 0.1% of Gadolinium [54] could improve our knowledge on the true value of $\sin^2\theta_{12}$ to 18% uncertainty [52]. The proposed/planned future experiments aiming to measure the very low energy pp flux coming from the sun are also expected to give a better measurement of the solar mixing angle [55]. However, even with very small experimental uncertainty of only 1%, they are not expected to reduce the uncertainty on $\sin^2\theta_{12}$ much better than $\sim 14\%$ at 3σ [41, 56]. The only type of experiment that would provide extremely good measurement of the solar mixing angle is a long baseline reactor experiment with its baseline tuned to a Survival Probability minimum [51], which is given by the condition $L_{min} \simeq 1.24~E/\text{MeV}~\text{eV}^2/\Delta m_{21}^2$. This type of experiment could be used to measure the mixing angle $\sin^2\theta_{12}$ down to $\sim 6\%$ at 3σ [41, 57].

The Dirac CP phase δ has a faint chance of being measured in the next generation superbeam experiments [48, 49], provided the true value of θ_{13} is not too small. However, performing an unambiguous measurement, and especially establishing a signal for CP violation, would probably need a beta beam facility or a neutrino factory [58]. The two Majorana phases are measurable only in processes which violate lepton number. At present the only such process which seems to be viable experimentally is neutrinoless double beta decay [59]. For detailed analyzes of how to extract information on Majorana phases from $0\nu\beta\beta$ we refer to [16, 18, 19, 20]. The bottom–line of this subject is that for given oscillation parameters the Majorana phases should not be too close to 0 or $\pi/2$ and the uncertainty on both the experimental and theoretical side (i.e., the nuclear matrix elements) should be small [16, 20]. In addition, the prospects of determining the Majorana CP phases increase with increasing solar neutrino mixing angle.

The quest for the limit on the absolute neutrino mass scale will witness attacks both via direct kinematical searches and cosmological measurements. The KATRIN [60] experiment, currently under construction in Germany, is scheduled to start taking data in 2008 and is sensitive to neutrino masses down to 0.2 eV. Further cosmological probes will test the sum of neutrino masses down to 0.1 eV within this decade [12]. Additional data sets and novel experiments can reduce this number by a factor of two [61].

Finally, the last piece of information needed to construct completely the neutrino mass matrix is the ordering of the neutrino states – the sign of Δm_{31}^2. If the sign was positive we would have a normal mass ordering, while for a negative sign we would have an inverted mass ordering. Typical approaches to identify $\text{sgn}(\Delta m_{31}^2)$ rely on using matter effects. Since the earth matter effect for modest baselines, and therefore modest matter densities, depends crucially on the size of the mixing angle θ_{13}, it becomes exceedingly difficult to study the mass hierarchy as θ_{13} becomes small. If θ_{13} was large, close to its current limit, there could be a small chance of measuring the mass hierarchy using the synergies between T2K and NOνA [62]. However, the measurement would still not be very unambiguous and one would need either a beta beam facility or a neutrino factory for the hierarchy determination [58]. Very large matter effects in the 1–3 channel are expected for supernova neutrinos. Therefore, a supernova neutrino signal could in principle be used to determine the sign of Δm_{31}^2 [63]. Resonant matter effects in the 1–3 channel are also encountered by atmospheric neutrinos as they cross large baselines in their passage through the earth. This

can be exploited to probe the neutrino hierarchy both in water Cerenkov and large magnetized iron calorimeter detectors [45, 64]. Recently some novel ways of probing the mass hierarchy requiring very precise measurements and using the "interference terms" between the different oscillation frequencies have been proposed [65] (see also [66]). However, it is understood that among all the neutrino parameters, the mass hierarchy determination is expected – together with the determination of the Majorana phases – to be the most challenging for the future experiments. In this respect the neutrinoless double beta decay is of some interest, since the question of distinguishing the neutrino mass hierarchy can be answered by $0\nu\beta\beta$. This issue will be discussed in the following Section.

3 Distinguishing the Neutrino Mass Schemes

In this Section we present the phenomenology of neutrinoless double beta decay in terms of the oscillation parameters Δm_{31}^2, Δm_{21}^2, $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$. In particular we will look how easy or difficult it would be to distinguish the different neutrino mass schemes if we have a signal or a significantly improved limit for $0\nu\beta\beta$.

For the NH scheme, for $m_1 \ll m_2 \ll m_3$ and therefore assuming that m_1 can be neglected, we have

$$\langle m \rangle^{\mathrm{NH}} \simeq \left| \sqrt{\Delta m_{21}^2}\, \sin^2\theta_{12}\, \cos^2\theta_{13} + \sqrt{\Delta m_{31}^2}\, \sin^2\theta_{13}\, e^{2i(\beta-\alpha)} \right| . \tag{13}$$

The maximal value, $\langle m \rangle_{\mathrm{max}}^{\mathrm{NH}}$, is achieved when both terms add up, which corresponds to $\alpha - \beta = \pm 2\pi, \pm\pi$ or 0. The minimal value of $\langle m \rangle$ corresponds to $\alpha - \beta = \pm\pi/2, 3\pi/2, 5\pi/2$ or $7\pi/2$, resulting in (partial) cancellation between the two terms in Eq. (13). For non–zero values of m_1 between 10^{-2} and 10^{-3} eV there can be complete cancellation resulting in a vanishing effective mass. This will be subject of Section 5.

For the IH scheme, assuming that $m_3 \ll m_1 < m_2$ and neglecting m_3, we have

$$\langle m \rangle^{\mathrm{IH}} \simeq \sqrt{|\Delta m_{31}^2|}\, \cos^2\theta_{13}\, \sqrt{1 - \sin^2 2\theta_{12}\, \sin^2\alpha} . \tag{14}$$

Here the maximal (minimal) value is obtained when $\alpha = 0, \pi$ or 2π ($\alpha = \pi/2, 3\pi/2, 5\pi/2$ or $7\pi/2$).

Finally, for the QD mass spectrum

$$\langle m \rangle^{\mathrm{QD}} \simeq m_0 \left| (\cos^2\theta_{12} + \sin^2\theta_{12}\, e^{2i\alpha})\cos^2\theta_{13} + e^{2i\beta}\, \sin^2\theta_{13} \right| . \tag{15}$$

If the terms proportional to $e^{2i\alpha}$ and $e^{2i\beta}$ add up (i.e., when α and β take values of $0, \pi$ or 2π) we have the maximal value of $\langle m \rangle$, which is then just m_0. On the other hand, the minimal $\langle m \rangle$ is achieved when α and β take values of $\pi/2, 3\pi/2, 5\pi/2$ or $7\pi/2$. We will use this to set limits on m_0 in Section 4.

Hence, for a given mass scheme, the effective mass depends on Δm_{31}^2, Δm_{21}^2, $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and on the Majorana phases (to be precise, on one of them, or on a combination

thereof). In case of QD the absolute neutrino mass scale m_0 plays a decisive role as well. In the case of NH the smallest neutrino mass could be extremely important in deciding the degree of cancellation between the different terms, as discussed above. The lack of knowledge of most of those parameters means that we can not give definite predictions for the value of $\langle m \rangle$ for any of the three extreme neutrino mass schemes. We can however give a range of $\langle m \rangle$ for NH, IH and QD, namely:

$$0.0 < \langle m \rangle < 0.007 \text{ eV} \qquad , \qquad \text{NH} \qquad (16)$$

$$0.006 \text{ eV} < \langle m \rangle < 0.065 \text{ eV} \qquad , \qquad \text{IH} \qquad (17)$$

$$0.41 \ (0.07) \text{ eV} < \langle m \rangle < 2.3 \ (0.4) \text{ eV} \quad , \qquad \text{QD} \qquad (18)$$

In order to obtain those values, we took the ranges of the oscillation parameters from Eqs. (7,8,9,11) and inserted them in the expressions for the maximal and minimal values for $\langle m \rangle$ in the three extreme schemes under consideration. For the case of QD we used the limit from the direct kinematical search [38, 39] and in brackets $m_0 < 0.4$ eV. Note that with the 3σ values the NH and IH cases slightly overlap. However, these ranges are expected to reduce remarkably in the future from more precise measurements of the solar and atmospheric neutrino parameters and from either a signal for a non–zero value of (or from better upper limits on) $\sin^2 \theta_{13}$. Nevertheless, already at the present stage the possibility of distinguishing the mass spectra from each other opens up. In particular, the case of deciding between NH and IH is the most interesting one. Distinguishing the QD spectrum from NH or IH will be at most a consistency check since the common neutrino mass of or above 0.2 eV will be probed by either direct searches or cosmology.

We present in Figure 1 the lines of constant maximal and minimal $\langle m \rangle$ for the cases of NH and IH, respectively, in the $\sin^2 \theta_{12} - \sin^2 \theta_{13}$ plane. We fixed the values of $\Delta m_{31}^2 = 0.002$ eV^2 and $\Delta m_{21}^2 = 8 \times 10^{-5}$ eV^2. The dotted (red) lines show the lines of constant minimum value of $\langle m \rangle$ for the IH scheme ($\langle m \rangle_{\text{min}}^{\text{IH}}$) and the solid (blue) lines give the lines of constant maximum value of $\langle m \rangle$ for the NH scheme ($\langle m \rangle_{\text{max}}^{\text{NH}}$). The Figure illustrates the well–known fact that $\langle m \rangle_{\text{min}}^{\text{IH}}$ has a strong $\sin^2 \theta_{12}$ dependence, while $\langle m \rangle_{\text{min}}^{\text{NH}}$ depends on both $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$. Also shown by the dot–dashed (black) line on the Figure are the current allowed values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ at the 3σ level, obtained from a 2 parameter fit of the global oscillation data [10][4]. As an example, one can see from the plot that if we know that the mass ordering is normal and assume that the smallest neutrino mass is negligible, values of the effective mass larger than roughly 0.0053 eV are incompatible with the currently allowed values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$. The same would be the case when we know that the mass ordering is inverted, assume that the smallest neutrino mass is negligible and the effective mass is larger (or smaller) than 0.023 (0.008) eV.

We can see from Eqs. (13) and (14) that the maximal and minimal values of $\langle m \rangle$ are crucially dependent on the values of the mass squared differences, especially on Δm_{31}^2. Since we are talking about a future measurement of the $0\nu\beta\beta$ signal, we take the expected

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[4]The 3σ limit $\sin^2 \theta_{13} < 0.044$ given in the Introduction and Section 2 was obtained in a 1 parameter fit of the global neutrino data and is therefore different from the one in Figure 1.

futuristic uncertainties mentioned in Section 2 on Δm_{31}^2 and Δm_{21}^2 to plot the iso–range of $\langle m \rangle_{\min}^{\text{IH}}$ and $\langle m \rangle_{\max}^{\text{NH}}$ in Figures 2 and 3. We can see clearly from the Figures that the uncertainty on the predicted value of $\langle m \rangle$ due to the uncertainty in the value of Δm_{31}^2 and Δm_{21}^2 would become very small in the next ten years. Therefore, in what follows, we will keep Δm_{31}^2 and Δm_{21}^2 fixed at $\Delta m_{31}^2 = 0.002$ eV^2 and $\Delta m_{21}^2 = 8 \times 10^{-5}$ eV^2.

Any future positive signal for $0\nu\beta\beta$ will be able to distinguish the IH scheme from the NH scheme if the measured value of $\langle m \rangle$ would be such that it could be explained by the predicted $\langle m \rangle$ for just one of the schemes and not the other. In other words, if the difference between the predicted values for $\langle m \rangle$ for the IH scheme and the NH scheme is larger than the error in the measured value of $\langle m \rangle$, then it would be possible (assuming that the smallest neutrino mass is negligible) to experimentally find the neutrino mass hierarchy from a measurement of $0\nu\beta\beta$. We therefore show the difference in the predicted value of $\langle m \rangle$ for the two schemes,

$$\Delta\langle m \rangle = \langle m \rangle_{\min}^{\text{IH}} - \langle m \rangle_{\max}^{\text{NH}} , \qquad (19)$$

in left–hand panel of Figure 4 as a function of $\sin^2\theta_{12}$ for 4 fixed values of $\sin^2\theta_{13}$ and in right–hand panel of Figure 4 as a function of $\sin^2\theta_{13}$ for 4 fixed values of $\sin^2\theta_{12}$. We see that $\Delta\langle m \rangle$ displays a strong dependence on $\sin^2\theta_{12}$, whereas the dependence on $\sin^2\theta_{13}$ is rather moderate. In fact, it holds that

$$\Delta\langle m \rangle = \frac{\sqrt{\Delta m_{\text{A}}^2}\,\cos^2\theta_{13}}{1 + \tan^2\theta_{12}} \left(1 - \tan^2\theta_{12} \left(1 + R + \tan^2\theta_{13} \right) - \tan^2\theta_{13} \right) , \qquad (20)$$

where R is the ratio of the solar and atmospheric Δm^2 defined in Eq. (12). Figure 4 and Eq. (20) demonstrate the well–known fact that distinguishing the normal and inverted hierarchy is easier for smaller values of $\sin^2\theta_{12}$. The values of $\Delta\langle m \rangle$, which are of the order 0.01 eV, represent a value of the maximal experimental uncertainty which an experiment should have in order to be able to distinguish NH from IH.

Up to here we neglected the complications which arise from the nuclear matrix elements (NME) involved in the calculation of the rate of $0\nu\beta\beta$. There is neither a consensus in the literature on how large this uncertainty is, nor if there really is a large uncertainty at all [67]. Let us therefore take a conservative point of view and analyze the situation as a function of the NME uncertainty. If there is indeed no problem with the NME, the statements given up to this point apply.

Given the possibility of large uncertainties on the values of the nuclear matrix elements, it is plausible to ask if one could possibly extract the true hierarchy of neutrino masses from a positive signal for $0\nu\beta\beta$ in the future. To that end, we look for the difference between $\langle m \rangle_{\min}^{\text{IH}}$ and $\langle m \rangle_{\max}^{\text{NH}}$ after including the uncertainties coming from the nuclear matrix elements. A way to perform such an analysis has been developed in [16], and here we follow a very similar approach: in order to parametrize the uncertainty coming from the nuclear matrix elements, we define the $0\nu\beta\beta$ decay rate Γ measured in any experiment such that

$$\sqrt{\Gamma} = x\,\langle m \rangle , \qquad (21)$$

where x contains all other factors involved in the $0\nu\beta\beta$ decay, including the NME. Thus for any measured Γ the measured value of $\langle m \rangle$ would be

$$\langle m \rangle_{\text{measured}} = \sqrt{\Gamma}/x \ . \tag{22}$$

If y is the smallest possible value for x, and if we assume that the uncertainty on x originates solely from the uncertainty on the NME and is a factor of z (with $z > 1$), then the range of x is given by $y < x < z\, y$. Therefore, the uncertainty on the measured value of $\langle m \rangle$ that we have due to the uncertainty on the NME is given by

$$\Delta \langle m \rangle_{\text{measured}} = \frac{z - 1}{z + 1} \ . \tag{23}$$

In order to be able to experimentally distinguish IH from NH with the help of a $0\nu\beta\beta$ measurement, we must have $\langle m \rangle_{\text{min}}^{\text{IH}} > \langle m \rangle_{\text{max}}^{\text{NH}}$ *after* including the uncertainty from the NME. Thus we need the condition

$$\langle m \rangle_{\text{min}}^{\text{IH}} - z \, \langle m \rangle_{\text{max}}^{\text{NH}} > 0 \tag{24}$$

to ascertain the neutrino mass hierarchy. In the left and right panel of Figure 5 we plot the difference $\langle m \rangle_{\text{min}}^{\text{IH}} - z \, \langle m \rangle_{\text{max}}^{\text{NH}}$ predicted as a function of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$, respectively, including the uncertainty coming from the NME. We show the plots assuming a range of values for the NME uncertainty z. As a function of $\sin^2 \theta_{12}$ the quantity $\langle m \rangle_{\text{min}}^{\text{IH}} - z \, \langle m \rangle_{\text{max}}^{\text{NH}}$ shows a similar behavior as for no uncertainty in the NME. The dependence on $\sin^2 \theta_{13}$ is rather weak for small z, but becomes larger with increasing z. From Figure 5 one can read of the maximal experimental uncertainty for a given z, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{12}$ for which one could still be able to distinguish the mass hierarchy. For instance, taking $\sin^2 \theta_{13}$ close to its current limit and assuming $z = 2$ and $\sin^2 \theta_{12} = 0.3$, we have $\langle m \rangle_{\text{min}}^{\text{IH}} - z \, \langle m \rangle_{\text{max}}^{\text{NH}} \simeq 0.01$ eV and the experimental uncertainty should not be larger than this value. For zero θ_{13} this value becomes larger by roughly 40%.

Up to now we considered the case of the smallest mass m_{sm} being zero, which would correspond to the neutrino mass matrix having a zero determinant, clearly a rather limited and special case. For a given set of oscillation parameters and the nuclear matrix elements uncertainty, it would be possible to distinguish between the NH and IH in a $0\nu\beta\beta$ experiment only if m_{sm} is less than a certain value. It is interesting to examine up to which values of the lightest neutrino mass one can distinguish NH from IH. Toward this exercise, we plot in Figure 6 the difference between the minimal value of $\langle m \rangle$ for IH and the maximal value of $\langle m \rangle$ for NH, $\langle m \rangle_{\text{min}}^{\text{IH}} - \langle m \rangle_{\text{max}}^{\text{NH}}$, as a function of the smallest neutrino mass m_{sm}. The NME uncertainty is also included in the analysis. For each of the 4 panels we assumed a certain "true" value of $\sin^2 \theta_{12}$ with an optimistic uncertainty of 6% [41, 57]. For $\sin^2 \theta_{13}$ we assumed that no positive signal is observed in any of the forthcoming experiments in the next ten years. Thus we allowed it to vary within 0 and 0.0025 [42]. We see from the Figure that for $\sin^2 \theta_{12} = 0.3$ for example, when $z = 1.5$ ($z = 1$, *viz.* no NME uncertainty) and $\Delta \langle m \rangle = 0.01$ eV, we can distinguish NH from IH if m_{sm} is smaller than 0.002 (0.004)

eV. Of course, the value of $\Delta\langle m \rangle$ obtained like this is again an upper limit on the experimental uncertainty which would still allow distinguishing NH from IH for a non–zero smallest mass. If $\sin^2\theta_{12} = 0.26$ the situation looks more promising. For the same value of $\Delta\langle m \rangle = 0.01$ eV we can distinguish NH from IH if $m_{\rm sm}$ is smaller than 0.005 (0.01) eV if $z = 1.5$ ($z = 1$). Other cases are easily read off Figure 6.

For a larger uncertainty in the value of $\sin^2\theta_{12}$ and/or for a larger true value of $\sin^2\theta_{13}$, the chances of distinguishing the IH from NH become worse and in addition only work for very small values of m_{sm}.

We see however that in principle values of $m_{\rm sm}$ up to ~ 0.01 eV are allowed in order to distinguish the normal from the inverted neutrino mass hierarchy.

4   Limit on the Neutrino Mass

Another interesting aspect of neutrinoless double beta decay is the possibility to set a limit on the absolute scale of the neutrino mass. The current upper value for the effective mass is somewhere between 0.3 and 1 eV, where this range of course has its origin in the NME uncertainty. The indicated values correspond to the QD mass spectrum, on which we wish to focus in this Section. As indicated in Section 3, for a given common mass scale m_0, the lowest possible effective mass can be written as

$$\langle m \rangle_{\rm min}^{\rm QD} = m_0 \left(|U_{e1}|^2 - |U_{e2}|^2 - |U_{e3}|^2 \right) = \frac{1 - \tan^2\theta_{12} - 2\,|U_{e3}|^2}{1 + \tan^2\theta_{12}} \ . \tag{25}$$

Hence, having a limit on the effective mass at hand, we can translate it into a limit on the neutrino mass. We write the experimental limit as

$$\langle m \rangle^{\rm exp} = z\,\langle m \rangle_{\rm min}^{\rm exp} \ , \tag{26}$$

where $\langle m \rangle_{\rm min}^{\rm exp}$ is defined as the limit on $\langle m \rangle$ obtained by using the largest available NME and $z \geq 1$ encodes again the NME uncertainty. Then, the limit on the neutrino mass reads

$$m_0 \leq z\,\langle m \rangle_{\rm min}^{\rm exp}\, \frac{1 + \tan^2\theta_{12}}{1 - \tan^2\theta_{12} - 2\,|U_{e3}|^2} \equiv z\,\langle m \rangle_{\rm min}^{\rm exp}\, f(\theta_{12}, \theta_{13}) \ . \tag{27}$$

We have introduced here a function $f(\theta_{12}, \theta_{13})$ in this expression, which separates the information available from neutrino oscillation experiments from the information coming from $0\nu\beta\beta$ and m_0. We think that this formulation might be helpful in understanding how neutrinoless double beta decay and the absolute neutrino mass scale are related. Currently the uncertainty on $f(\theta_{12}, \theta_{13})$ is around 50%, $1.9 < f(\theta_{12}, \theta_{13}) < 5.6$. It is expected to reduce to $\sim 21\%(\sim 9)$ % at 3σ if a low energy pp solar neutrino experiment (reactor experiment at the survival probability minimum) would be built. The uncertainty depends only little on the value of θ_{13}. From the current limit on the effective mass, $\langle m \rangle \leq 0.35\,z$ eV, with the accepted value of $z \simeq 3$ for ^{76}Ge (see [20]), and our current knowledge of $f(\theta_{12}, \theta_{13})$, we can set a limit on m_0 of 5.6 eV, clearly weaker than the limit from tritium

experiments. Only for close to vanishing NME uncertainty (i.e., $z = 1$) we can reach values of $m_0 \lesssim 2$ eV which are then comparable to ones from direct searches.

In Figure 7 we show the iso–contours of m_0 predicted for the QD mass spectrum (cf. Eq. (27)) in the $\sin^2 \theta_{12} - \sin^2 \theta_{13}$ plane. For each of the iso–contours we have assumed an illustrative value of $z \langle m \rangle^{\text{exp}}_{\text{min}} = 0.1$ eV. Also shown is the current 3σ allowed area of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$. From the Figure we can see that for a measured value of $z \langle m \rangle^{\text{exp}}_{\text{min}} = 0.1$ eV, the constraint on the common mass scale for a QD spectrum would be: 0.2 eV $\lesssim m_0 \lesssim 0.6$ eV. For any other measured value of $z \langle m \rangle^{\text{exp}}_{\text{min}}$ corresponding to a QD mass scheme, we can obtain the corresponding constraints on m_0 by scaling the above limit suitably, i.e., for $z \langle m \rangle^{\text{exp}}_{\text{min}} = 0.2$ eV we would have 0.4 eV $\lesssim m_0 \lesssim 1.2$ eV. The limits on m_0 given above are of course with our existing knowledge about θ_{12} and θ_{13}. With reduction of the range of allowed values of θ_{12} and θ_{13}, especially θ_{12}, the constraints on m_0 are expected to reduce substantially, as discussed above. For instance, if $f(\theta_{12}, \theta_{13})$ was known with an uncertainty of 20%, say $2.7 < f(\theta_{12}, \theta_{13}) < 4.0$, then we would have for $z \langle m \rangle^{\text{exp}}_{\text{min}} = 0.1$ (0.2) eV that 0.3 eV $\lesssim m_0 \lesssim 0.4$ eV $(0.6$ eV $\lesssim m_0 \lesssim 0.8$ eV$)$. Of course, if we have no signal for $0\nu\beta\beta$, but just an upper limit on $z \langle m \rangle_{\text{min}}$, we have no longer an allowed range on m_0, but an upper limit corresponding to the largest value in the range. From the examples given above, one can note that for the QD mass spectrum, a measurement or a better constraint on $\langle m \rangle$ will lead to a stronger limit on the absolute neutrino mass scale compared to the current limit from direct kinematical searches.

5 Implications of a Vanishing effective Mass

It is well–known that not only a measurement but also a non–measurement of neutrinoless double beta decay has significant influence on our knowledge of the unknown neutrino parameters. In this Section we wish to discuss a rarely studied subject, namely the impact that a very small upper limit on $\langle m \rangle$ would have[5]. An exactly vanishing $\langle m \rangle$ would correspond to a texture zero in the neutrino mass matrix in the charged lepton basis – certainly an interesting feature. However, to prove/observe an exactly vanishing effective mass is a formidable task.

In case of an inverted hierarchy we know that there is a lower limit of $\langle m \rangle$ which is — when using current 3σ values of the oscillation parameters — given by roughly 0.006 eV, see Eq. (17). Let us assume that neutrinos are Majorana particles, have an inverted ordering and that the effective mass takes has an upper limit of order 0.01 eV. This is a situation which might arise if we know from some other independent experiment that $\text{sgn}(\Delta m^2_{\text{A}}) = -1$ and the next generation $0\nu\beta\beta$–experiments do not find a signal corresponding to a non–vanishing $\langle m \rangle$, but give an upper limit on $\langle m \rangle$ still above the theoretical limit $\langle m \rangle^{\text{IH}}_{\text{min}}$. Then we can infer from Eq. (14) the values of $\sin^2 \theta_{12}$ and the Majorana phase α which are still compatible with the data. In Fig. 8 we display the result for an experimental upper limit on $\langle m \rangle$ of 0.04, 0.03, 0.02, 0.01 eV, taking $m_3 = 0$, $\theta_{13} = 0$ and $\Delta m^2_{31} = 0.002$ eV^2. We checked that the results are rather stable when we depart from

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[5]For a related earlier analysis, see [68].

these values within their current uncertainty. The current 3σ values of $\sin^2\theta_{12}$ are also indicated in the Figure. Allowed is the area to the right of the respective curves. For instance, if $\langle m \rangle < 0.02$ eV and $\sin^2\theta_{12} = 0.3$ then α has to lie between 1.2 and 1.9, or $\alpha \simeq \pi/2 \pm 0.4$. Alternatively, if $\langle m \rangle < 0.02$ eV and $\sin^2\theta_{12} < 0.28$, then the IH case is ruled out.

Figure 9 shows a similar analysis for the case of NH with a smallest mass $m_1 = 0$. In our parametrization, as can be seen from Eq. (13), it is the combination $\beta - \alpha$ which governs the destructive interference which leads to very small or zero $\langle m \rangle$. We took very small upper limits on $\langle m \rangle$ of 0.004, 0.003, 0.002 and 0.001 eV (hence this discussion will not become realistic within the next 10 years) and chose $m_1 = 0$, $\sin^2\theta_{13} = 0.04$ $\Delta m_{21}^2 = 8 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 = 0.002$ eV^2. Note that for very small values of θ_{13}, corresponding to $\sin^2\theta_{13} \ll R\sin^2\theta_{12} \sim 0.01$, the dependence on this combination of phases drops. In this part of the Figure it is the left of the respective curve which is allowed. For instance, for $\langle m \rangle \lesssim 0.002$ eV and a rather large value of $\sin^2\theta_{12} = 0.4$ the phases have to be very close to $\pi/2$, namely $\beta - \alpha \simeq \pi/2 \pm 0.2$. Alternatively, if $\langle m \rangle < 0.001$ eV and $\sin^2\theta_{12} > 0.33$, then the NH case is ruled out.

As mentioned earlier in Section 3 we note here that in the case of NH, $\langle m \rangle$ could have a sizable dependence on the smallest neutrino mass state m_1. We plot therefore in Fig. 10 the same as in Fig. 9 but for a smallest neutrino mass of $m_1 = 0.005$ eV. We took again $\sin^2\theta_{13} = 0.04$, $\Delta m_{21}^2 = 8 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 = 0.002$ eV^2 and chose the same four upper limits on $\langle m \rangle$ as above. With a non–vanishing m_1 Eq. (13) is modified to

$$\langle m \rangle \simeq \left| m_1 \cos^2\theta_{12} \cos^2\theta_{13} + \sqrt{m_1^2 + \Delta m_\odot^2}\, \sin^2\theta_{12} \cos^2\theta_{13}\, e^{2i\alpha} + \sqrt{\Delta m_A^2}\, \sin^2\theta_{13}\, e^{2i\beta} \right| \quad (28)$$

and thus it is no longer $\beta - \alpha$ but both α and β which play a role. In Figure 10 we fixed $\alpha = \pi/2$ which gives a negative sign for the second term in Eq. (28). We indicate the allowed (disallowed) values of the parameter regions in the Figure with A (D). If $\langle m \rangle < 0.004$ eV and $\langle m \rangle < 0.003$ eV the disallowed areas are already outside the currently allowed 3σ region of $\sin^2\theta_{12}$ for α fixed at $\pi/2$. To show the dependence of our results on α, we plot in Figure 11 the allowed areas in the α–β plane for four different fixed values of $\sin^2\theta_{12}$, keeping $\sin^2\theta_{13}$ fixed at 0.04 and for $m_1 = 0.005$ eV. Allowed are the areas within the respective curves.

An interesting aspect is when θ_{13} vanishes but m_1 is non–zero. Then the effective mass is a function of the phase α alone:

$$\langle m \rangle \simeq \left| m_1 \cos^2\theta_{12} + \sqrt{m_1^2 + \Delta m_\odot^2}\, \sin^2\theta_{12}\, e^{2i\alpha} \right| . \quad (29)$$

In Figure 12 we show the areas in the $\sin^2\theta_{12}$–α plane that would be allowed if we had an upper limit on $\langle m \rangle$ of 0.001 eV (black line), 0.002 eV (red line), 0.003 eV (green line) and 0.004 eV (blue line). The areas outside the closed curves would be disallowed. We can see that for a limit of $\langle m \rangle = 0.001$ eV, all values of α could be possible while for $\langle m \rangle = 0.004$ eV,

only the range $\pi/2 \pm 0.67$ is allowed. This allowed range of α is not expected to improve with the reduction of the uncertainty on $\sin^2\theta_{12}$ since its θ_{12} dependence, as can be seen from the Figure, is very weak.

For values of the smallest mass of $m_1 \simeq 0.005$ eV and $\sin^2\theta_{12} \simeq 0.3$ both terms in Eq. (29) are of roughly the same magnitude (i.e., $m_1 \simeq m_2$) and we could write

$$\langle m \rangle \simeq m_{\rm NH} \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \alpha} \ , \ \text{where} \ m_{\rm NH} \equiv \sqrt{m_1^2 + \Delta m_\odot^2} \sim 0.01 \ \text{eV}.$$

Note the similarity of this equation with Eq. (14) in case of IH.

To sum up, in certain cases it might be possible to significantly constrain the allowed values of the Majorana phases. Moreover, the allowed values of the solar neutrino oscillation parameter θ_{12} can be comparable to its current known range.

6 Neutrinoless Double Beta Decay, Future Neutrino Oscillation Data and the Identification of the Neutrino Mass Matrix

In this Section we wish to give a summary of typical neutrino mass matrices available in the vast literature. Predictions for and correlations between the neutrino observables are implied by each of the candidates and can be used to distinguish them. In particular, knowledge of the neutrino mass spectrum, R, U_{e3}, θ_{23} and of course the effective mass are then very helpful for identifying the correct mass matrix. A complete study of all possibilities is (and maybe can) not performed, however, we feel that the majority of the models will produce at the end of the day a mass matrix which will more or less correspond to one of the examples given here. For instance, there are many symmetries which eventually lead to a $\mu\tau$ symmetric mass matrix [69], see for instance Refs. [70] for some examples. For more models and details we refer to the excellent reviews available [71]. We should remark that for the given examples, unless otherwise stated, the charged lepton mass matrix m_ℓ is diagonal, i.e., for the matrix U_ℓ diagonalizing $m_\ell m_\ell^\dagger$ holds that $U_\ell = \mathbb{1}$, and therefore the PMNS matrix is just $U = U_\ell^\dagger U_\nu = U_\nu$, where U_ν diagonalizes m_ν. Further note that most correlations focus on the quantities $\langle m \rangle$, R, θ_{23}, $|U_{e3}|$ or θ_{12}. Interplay with the parameters describing CP violation is rarely studied.

Let us focus first on the normal hierarchy, for which there is a very successful and well–known texture, namely [72]

$$m_\nu = m_0 \begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix} . \tag{30}$$

Here a, b, d, e, f, g are complex parameters of order one and ϵ is a real and small parameter typically of order of the Cabibbo angle $\lambda \simeq 0.22$. Such a mass matrix preserves at leading order the lepton charge L_e and can be obtained already by simple models based on $U(1)$ charges or through sequential right–handed neutrino dominance [73]. A correlation resulting from Eq. (30) is

$$\langle m \rangle = c_1 \sqrt{\Delta m_{\rm A}^2} \, |U_{e3}|^2 \;\; {\rm with} \;\; |U_{e3}| = c_2 \, R \; , \tag{31}$$

where $c_{1,2}$ are functions of the order one parameters and naturally also of order one. With the parameters in the $\mu\tau$ block unspecified, the atmospheric neutrino mixing angle typically deviates sizable from its maximal value $\pi/4$, i.e., $\theta_{23} = \pi/4 - c_3 \sqrt{R}$.

The discussion is also possible in the context of $\mu\tau$ symmetric mass matrices [69, 70]. Since their predictions include maximal θ_{23} and zero θ_{13}, one is interested in breaking schemes of the symmetry. Breaking the symmetry in the e sector of m_ν leads to the same correlation [74] as in Eq. (31) but atmospheric neutrino mixing is much closer to $\pi/4$. On the other hand, breaking the $\mu\tau$ symmetry in the $\mu\tau$ sector of m_ν leads to [74]

$$\langle m \rangle \simeq c_1 \sqrt{\Delta m_{\rm A}^2} \, |U_{e3}| \; , \tag{32}$$

where $|U_{e3}|$ is now very small, $|U_{e3}| = c_2 \, R$, and the deviation from maximal atmospheric neutrino mixing is again sizable, i.e., of order $\sqrt{R}$. Another interesting breaking scenario of $\mu\tau$ symmetric mass matrices is when the $e\mu$ element is not identical to, but the complex conjugate of the $e\tau$ element [75]. Then one finds that $U_{e3} \neq 0$ but ${\rm Re}\, U_{e3} = 0$, i.e., there is maximal CP violation with $\delta = \pm\pi/2$. A special case of $\mu\tau$ symmetric mass matrices, for which similar statements will apply, is given when $\tan^2 \theta_{12} = 0.5$, which then corresponds to the so–called tri–bimaximal mixing scheme [76]. "Bimaximal" scenarios [77] with their prediction $\tan^2 \theta_{12} = 1$ typically require contributions from the charged leptons, see below.

Perturbing a zeroth order mass matrix that leads to zero U_{e3} and maximal θ_{23} with a random matrix containing small entries of the same order ϵ [78], will lead to a semi–hierarchical mass spectrum (i.e., $m_2 > m_1$ and not $m_2 \gg m_1$) with a somewhat larger $\langle m \rangle$ and also to sizable deviations both from $U_{e3} = 0$ and $\theta_{23} = \pi/4$ [78].

Other candidates studied frequently in the literature are minimal models in the sense of having zeros in m_ν [79] or a minimal number of parameters in a see–saw context [80].

Table 1 summarizes our collection of the typical mass matrices for the normal mass hierarchy. Once we know that the neutrino mass spectrum is hierarchical, an information that could be provided by a negative search for $0\nu\beta\beta$ in future experiments and knowing that ${\rm sgn}(\Delta m_{31}^2) = +1$, we can sort out the correct mass matrix when we have precise information on the oscillation parameters.

In case of the inverted hierarchy, summarized in Table 2, the most stable candidate "theory" corresponds to a mass matrix conserving the flavor charge $L_e - L_\mu - L_\tau$ [81].

Such a matrix is given by

$$m_\nu = m_0 \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}, \tag{33}$$

predicting one massless neutrino, zero θ_{13}, zero $\langle m \rangle$, maximal *solar* neutrino mixing with $\Delta m_\odot^2 = 0$ and $\theta_{23} = \theta$. Barring extreme fine–tuning, it is impossible to perturb the structure of Eq. (33) such that the measured values of θ_{12} and $\Delta m_\odot^2$ are both accommodated in accordance with the data. Typically, after breaking it will hold that $\tan^2\theta_{12} \simeq 1 \pm R/2$ ($\sin^2\theta_{12} \simeq \frac{1}{2} \pm R/4$), whereas experimentally $\sin^2\theta_{12} \simeq \frac{1}{2} - \sqrt{R}$ is required. An exception is a see–saw model in which the perturbations at high energy have the same order of magnitude as the terms allowed by the symmetry, see [82]. In that example, U_{e3} and the smallest mass state remain zero. Another way out is to take corrections from the charged lepton sector into account [83]. The matrix U_ℓ, which diagonalizes $m_\ell m_\ell^\dagger$, is multiplied from the left to U_ν, which is the matrix diagonalizing m_ν from Eq. (33). The latter has to be perturbed in order to generate a non–zero $\Delta m_\odot^2$. Given the observation that the deviation from maximal solar neutrino mixing is determined by the Cabibbo angle, i.e., $\pi/4 - \theta_\odot \simeq \lambda \simeq 0.22 \simeq \theta_C$ [84], one assumes a CKM–like structure of U_ℓ [85][6]. The typical result of the described Ansatz lies in a correlation between the solar neutrino mixing and U_{e3}, namely $\sin^2\theta_{12} \simeq \frac{1}{2} - \cos\phi \cot\theta_{23} |U_{e3}|$, where ϕ is a phase appearing in m_ν, which can be given by the Dirac phases as measurable in oscillation experiments. This depends however on the breaking in m_ν. Nevertheless, both $|U_{e3}|$ and $\tan^2\theta_{12}$ are expected to be close to their current upper limits. Assuming minimal breaking in m_ν [83] one can show that there are remarkable correlations between the observables, such as $\langle m \rangle \simeq \sqrt{\Delta m_A^2} \left| \cos 2\theta_{12} + 4i\, J_{CP}/\sin^2\theta_{23} \right|$, where J_{CP} is the Jarlskog invariant for CP violating effects in neutrino oscillations, which is proportional to $|U_{e3}|/4$. This represents one of the few examples where the CP phases are part of the predicted correlations, depending however on the details of the model. The correlation $\tan^2\theta_{12} \simeq 1 - 4\,|U_{e3}|$ is however independent of the breaking in m_ν and relies only on bi–large U_ν [85]. Hence, the two discussed possibilities incorporating $L_e - L_\mu - L_\tau$ predict values for $\langle m \rangle$ of similar size but predict either zero or large U_{e3}.

There is another zeroth order scheme for the inverted hierarchy, namely

$$m_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1/2 & 1/2 \\ \cdot & \cdot & 1/2 \end{pmatrix} . \tag{34}$$

Zero U_{e3} and maximal θ_{23} are predicted and the two leading mass states have the same sign. One can perturb this structure [78] and the result is that both U_{e3} and $\theta_{23} - \pi/4$ are

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[6]Note that the so–called Quark–Lepton–Complementarity, i.e., the exact relation $\theta_{12} + \theta_C = \pi/4$ is a special case of this procedure.

very small quantities, but the effective mass is larger than in case of matrices based on the conservation of $L_e - L_\mu - L_\tau$.

The dependence of $\langle m \rangle$ on U_{e3} goes from none to sizable to little in the three cases discussed, cf. with Table 2. So, if we know that the mass ordering is inverted, i.e., $\text{sgn}(\Delta m_{31}^2) = -1$ and the effective mass is of order $\sqrt{\Delta m_A^2}$, then we basically only have to distinguish between $L_e - L_\mu - L_\tau$ from Eq. (33) and the matrix given in Eq. (34). This can be done by knowing how to close $\langle m \rangle$ is to $\sqrt{\Delta m_A^2}$. More precision data on U_{e3} and θ_{23} will fix the details of the model.

Now we turn to quasi–degenerate neutrinos. Usually, in such scenarios both the normal and the inverted mass ordering can be accommodated. Typical examples are given in Table 3. There are matrices with two zero entries which are compatible with quasi–degenerate neutrinos and which predict an interesting interplay of observables [79]. Anarchical mass matrices [86] do in general not generate extreme mixing angles and therefore typically result in large deviations from zero U_{e3} and maximal θ_{23}, i.e., of order R.

Possibilities to obtain QD neutrinos are models based on flavor democracy [87], of which we quote in Table 3 one interesting recent and typical example [88]. In such models both U_ν and U_ℓ are required in order to reproduce the neutrino data, resulting in a dependence of the neutrino observables on the charged lepton masses. This makes the predictions also a function of the democracy breaking scenario, therefore somewhat model–dependent. However, the smallness of $|U_{e3}|$ in such models can be attributed to the small ratios of the charged lepton masses.

One can also upgrade a hierarchical mass spectrum (e.g. resulting from sequential dominance) to a quasi–degenerate one by adding a term proportional to the unit matrix (e.g. connected to a $SO(3)$ symmetry) to it [89]. The neutrino mixing angles display approximately the same behavior as in the NH case (i.e., as from Eq. (30)), however, the effective mass is close to the common mass scale, i.e., there is little cancellation in $\langle m \rangle$. Moreover, the larger m_0 the smaller the phases [89].

Some attention has recently been caught by a model based on the discrete symmetry A_4 [90]. Applying the most general SUSY threshold corrections to the resulting mass matrix leads to predictions such as very large atmospheric mixing, purely imaginary U_{e3} with an absolute value of order $\sqrt{R}$ and $\langle m \rangle = m_0$. Similarly, one can use a simple Abelian symmetry corresponding to the conservation of the flavor symmetry $L_\mu - L_\tau$ [91] and perturb the resulting mass matrix[7]

$$
m_\nu = m_0 \begin{pmatrix} \cos\theta & 0 & 0 \\ \cdot & 0 & \sin\theta \\ \cdot & \cdot & 0 \end{pmatrix} ,
\tag{35}
$$

to which the A_4 model corresponds when $\theta = \pi/4$. The deviations from zero U_{e3} and maximal θ_{23} are small and inverse proportional to m_0^2. Note that interestingly most of the

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[7]Note that this matrix is a special case of $\mu\tau$ symmetry.

models presented here have $\langle m \rangle \simeq m_0$, i.e., the mass parameters hopefully measured in direct laboratory searches and in neutrinoless double beta decay experiments should be almost identical.

Another zeroth order scheme for quasi–degenerate neutrinos corresponds to a matrix of the form [92]

$$
m_\nu = m_0 \begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ \cdot & (1+\eta)/2 & (1+\eta)/2 \\ \cdot & \cdot & (1+\eta)/2 \end{pmatrix} .
\tag{36}
$$

Exact bimaximal mixing is predicted and breaking of the matrix is also required to generate splittings between the mass states. Similar statements as for the matrix Eq. (33) hold, i.e., without extreme fine–tuning it is impossible to generate deviations from maximal solar neutrino mixing sizable enough not to be in conflict with the data. Hence, the contribution from the charged lepton sector are required, which will lead to similar correlations as for $L_e - L_\mu - L_\tau$ discussed above, such as $\tan^2 \theta_{12} \simeq 1 - 4\, |U_{e3}|$. The correlation between the other observables depends strongly on the breaking. Since however the ee element is zero in Eq. (36), the effective mass is expected to be small, i.e., $\langle m \rangle \simeq m_0 \cos 2\theta_{12}$.

Hence, the identification of the mass matrix in case of a QD spectrum can be performed when information also from direct kinematical or from cosmological measurements is available. With the expected future precision data the identification of the mass matrix should also be possible.

To sum up this Section, a complete determination of the all neutrino observables and consequent identification of correlations will be able to discriminate between the various possible models. At least some of the possible candidates will be ruled out. Note finally that the values of the two Majorana phases, whose determination turned out to be an extremely challenging task, is not really required in order to identify the mass matrix.

7   Conclusions

We have analyzed some aspects of the connection between neutrinoless double beta decay, the neutrino mass scale and neutrino oscillation parameters. In particular, we concentrated on the question whether the expected future precision on the oscillation parameters simplifies certain important consequences of a measurement of, or improved upper limit on, $0\nu\beta\beta$. We first summarized the current situation of the determination of the nine physical parameters of the neutrino mass matrix and the prospects of improving our knowledge about them with future experiments. Then we analyzed how $0\nu\beta\beta$ can help in distinguishing the normal mass ordering from the inverted one. We included the nuclear matrix element uncertainty in the analysis and pointed attention to the fact that distinguishing NH from IH depends on the value of the smallest neutrino mass $m_{\rm sm}$. We analyzed this

point and found that in principle values of $m_{\rm sm} = 0.01$ eV are allowed, where of course the issue of the nuclear matrix elements can significantly spoil this possibility.

Then we investigated inasmuch $0\nu\beta\beta$ can be used to set a limit on the neutrino mass scale m_0 in case of a quasi–degenerate spectrum. We argued how the information from oscillation data and from $0\nu\beta\beta$ can be separated. Current limits on m_0 are weaker than direct ones but can be significantly improved with future $0\nu\beta\beta$–experiments and more precise knowledge of the oscillation parameters $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$.

Next we studied the implications of a very small, or even vanishing effective mass. Still insisting that neutrinos are Majorana particles, one can put in this case interesting constraints on parameters, in particular on $\sin^2\theta_{12}$ and the Majorana phases.

Finally, we tried to perform a scan through the literature and identified neutrino mass matrices, which tend to arise frequently in many different models. We listed their predictions and correlations and argued how future precision date of the oscillation parameters and information from neutrinoless double beta decay can help to identify the true neutrino mass matrix or at least to sort out many unsuccessful ones.

Acknowledgments

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Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}$	Ref. [72] e.g., simple $U(1)$, broken L_e sequential dominance	$\langle m \rangle = c_1\,\sqrt{\Delta m_{\rm A}^2}\,\|U_{e3}\|^2$ $\|U_{e3}\| = c_2\,\sqrt{R}, \ \theta_{23} = \frac{\pi}{4} - c_3\,\sqrt{R}$ normal hierarchy
$\begin{pmatrix} a\,\epsilon & b\,\epsilon & d\,\epsilon \\ \cdot & 1+f\,\epsilon & 1+g\,\epsilon \\ \cdot & \cdot & 1+h\,\epsilon \end{pmatrix}$	Ref. [78] perturbed m_ν^0 broken L_e	$\langle m \rangle = \frac{\sqrt{\Delta m_{\rm atm}^2}}{2}\,(1 + c_1\,\|U_{e3}\|)$ $\|U_{e3}\| = c_2\,\sqrt{R}, \ \theta_{23} = \frac{\pi}{4} - c_3\,\sqrt{R}$ normal ordering $m_2 > m_1$
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & 1+\epsilon & 1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$	Ref. [74] $\mu\tau$ symmetry broken in e sector	$\langle m \rangle = c_1\,\sqrt{\Delta m_{\rm A}^2}\,\|U_{e3}\|^2$ $\|U_{e3}\| = c_2\,\sqrt{R}, \ \theta_{23} = \frac{\pi}{4} - c_3\,R$ normal hierarchy
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & b\,\epsilon \\ \cdot & 1+d\epsilon & 1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$	Ref. [74] $\mu\tau$ symmetry broken in $\mu\tau$ sector	$\langle m \rangle = c_1\,\sqrt{\Delta m_{\rm A}^2}\,\|U_{e3}\|$ $\|U_{e3}\| = c_2\,R, \ \theta_{23} = \frac{\pi}{4} - c_3\,\sqrt{R}$ normal hierarchy
$\begin{pmatrix} 0 & 0 & \epsilon \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	Ref. [79] 2 zeros very similar to $m_{ee} = m_{e\tau} = 0$	$\langle m \rangle = 0$ $\|U_{e3}\| = \sqrt{\frac{R}{\cos 2\theta_{12}}}\,\frac{\sin 2\theta_{12}}{2\tan\theta_{23}}$ normal hierarchy
$\begin{pmatrix} a^2\,\epsilon & a\sqrt{1-a^2}\,\epsilon & 0 \\ \cdot & b^2 + (1-a^2)\,\epsilon & b\,\sqrt{1-b^2} \\ \cdot & \cdot & 1-b^2 \end{pmatrix}$	Ref. [80] minimal see–saw	$\langle m \rangle = \sqrt{\Delta m_\odot^2}\,\sin^2\theta_{12}$ $\|U_{e3}\| = \sqrt{R}/2\,\sin 2\theta_{12}\,\tan\theta_{23}$ normal hierarchy, $m_1 = 0$

Table 1: Typical and popular neutrino mass matrices predicting the normal mass hierarchy. Given are some comments about their origin and order of magnitude predictions for correlations between the observables. The parameter ϵ is usually small, i.e., mostly of order $\lambda \simeq 0.22$ and the parameters $a, b, d, e, f, c_{1,2,3}$ are of order one.

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} 1+a\,\epsilon & b\,\epsilon & d\,\epsilon \\ \cdot & 1/2+f\,\epsilon & 1/2+g\,\epsilon \\ \cdot & \cdot & 1/2+h\,\epsilon \end{pmatrix}$	Ref. [78] perturbed m_ν^0	$\langle m \rangle = \sqrt{\Delta m_{\rm A}^2}\,(1 + c_1\,\|U_{e3}\|)$ $\|U_{e3}\| = c_2\,R, \ \theta_{23} = \frac{\pi}{4} - c_3\,R$ inverted hierarchy
$\begin{pmatrix} 0 & a & b \\ \cdot & \epsilon^2 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	Ref. [83] broken $L_e - L_\mu - L_\tau$ and $U_\ell \sim V_{\rm CKM}$	$\langle m \rangle = \sqrt{\Delta m_{\rm A}^2}\,\left\|\cos 2\theta_{12} + 4i/\sin^2\theta_{23}\,J_{CP}\right\|$ $\tan^2\theta_{12} = 1 - 4\cos\delta\,\cot\theta_{23}\,\|U_{e3}\|$ inverted hierarchy

Table 2: Same as Table 1 for the inverted mass hierarchy. We have $J_{CP} = 1/8 \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$ with the neutrino oscillation CP phase δ. The correlations of the last row depend on the breaking of the mass matrix. For more complicated breaking, the phase δ will be some combination of phases.

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix} + \begin{array}{c} \text{sequential} \\ \text{dominance} \end{array}$	Ref. [89] type II see–saw upgrade	$\langle m \rangle \simeq m_0$ $|U_{e3}| = c_1 \sqrt{R}, \; \theta_{23} = \frac{\pi}{4} - c_2 \sqrt{R}$ phases shrink with m_0
$\begin{pmatrix} \delta & -1/\sqrt{2} & (1-\epsilon)/\sqrt{2} \\ \cdot & (1+\eta)/2 & (1+\eta-\epsilon)/2 \\ \cdot & \cdot & (1+\eta-2\epsilon)/2 \end{pmatrix}$	Ref. [92] $\epsilon \leftrightarrow$ radiative corrections	$\langle m \rangle \sim m_0 \cos 2\theta_{12}$ $\tan^2 \theta_{12} \simeq 1 - 4\, c_1\, |U_{e3}|$ (charged leptons required)
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix}$	Ref. [90] A_4 plus threshold corrections	$\langle m \rangle = m_0 \geq 0.3$ eV Re $U_{e3} = 0, \; |U_{e3}| = c_1 \sqrt{R}$ $\theta_{23} \simeq \pi/4$
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & -1 \\ \cdot & \cdot & 0 \end{pmatrix}$	Ref. [91] $L_\mu - L_\tau$ plus perturbations	$\langle m \rangle = m_0\, (1/\sqrt{2} + c_1\, |U_{e3}|)$ $|U_{e3}| = c_2\, \Delta m_{\rm A}^2/m_0^2 \lesssim 0.1$ $\theta_{23} = \pi/4 - c_3\, |U_{e3}|$
$\begin{pmatrix} a & \epsilon & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & d \end{pmatrix}$	Ref. [79] 2 zeros similar to $m_{e\mu} = m_{\tau\tau} = 0$ and $m_{e\mu} = m_{\mu\mu} = 0$ and $m_{e\tau} = m_{\tau\tau} = 0$	$\langle m \rangle \simeq m_0 \simeq \sqrt{\left| \frac{\Delta m_{\rm atm}^2 \tan^4 \theta_{23}}{1 - \tan^4 \theta_{23}} \right|}$ $R \simeq \frac{1 + \tan^2 \theta_{12}}{\tan \theta_{12}} \tan 2\theta_{23}$ Re U_{e3} $\Rightarrow \theta_{23} \neq \pi/4$ and Re $U_{e3} \simeq 0$
$\begin{pmatrix} a & \epsilon^2 & b\,\epsilon \\ \cdot & 0 & d \\ \cdot & \cdot & 0 \end{pmatrix}$	Ref. [79] 2 zeros	$\langle m \rangle \simeq m_0 \simeq \sqrt{\Delta m_{\rm A}^2} \cot 2\theta_{12} / \cos \delta$ Re $U_{e3} \simeq \cot 2\theta_{12} \cot 2\theta_{23}$
$r_\nu \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + c_\nu \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix}$	Ref. [88] $S(3)_L \times S(3)_R$ democracy	$\langle m \rangle \simeq m_0$, requires $r_\nu \ll 1$ $|U_{e3}| \simeq \sqrt{m_e/m_\mu}$, θ_{23} close to bound depends on $m_{e,\mu,\tau}$ and breaking
$\begin{pmatrix} a & b & d \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}$	Ref. [86] anarchy	$|U_{e3}|$ close to upper bound, θ_{23} close to bound extreme hierarchy unlikely

Table 3: Same as Table 1 for the quasi–degenerate mass spectrum. The quantity measurable in direct mass measurements is denoted m_0.
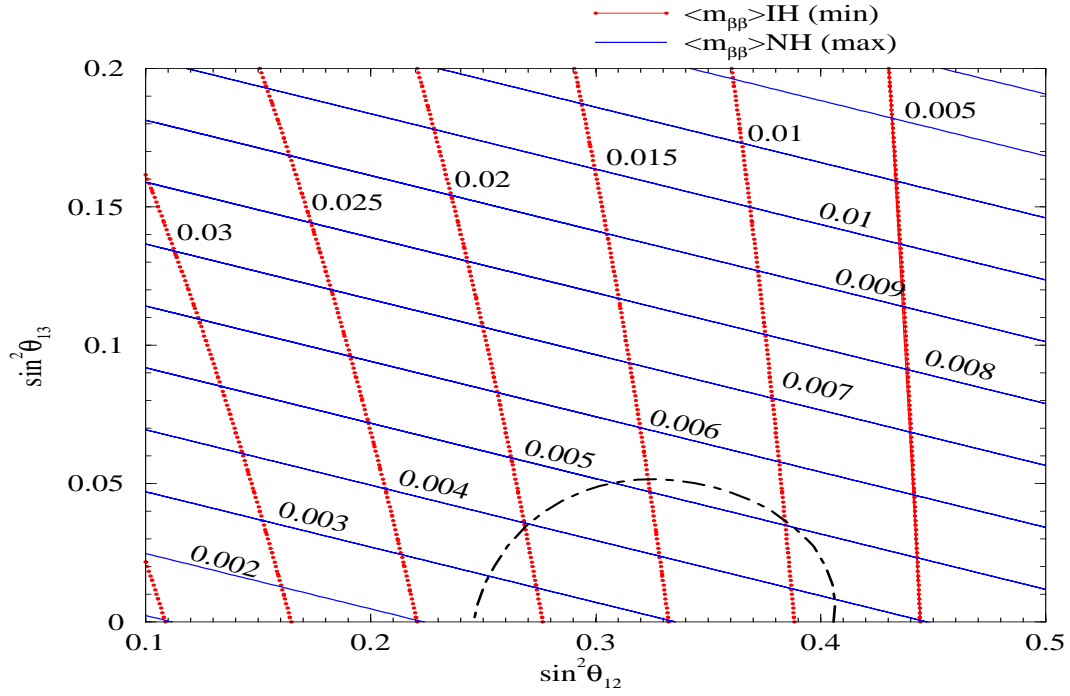
Figure 1: Lines of constant maximum value of $\langle m \rangle$ (in eV) for the NH scheme ($\langle m \rangle_{\mathrm{max}}^{\mathrm{NH}}$) and of constant minimum value of $\langle m \rangle$ for the IH scheme ($\langle m \rangle_{\mathrm{min}}^{\mathrm{IH}}$).
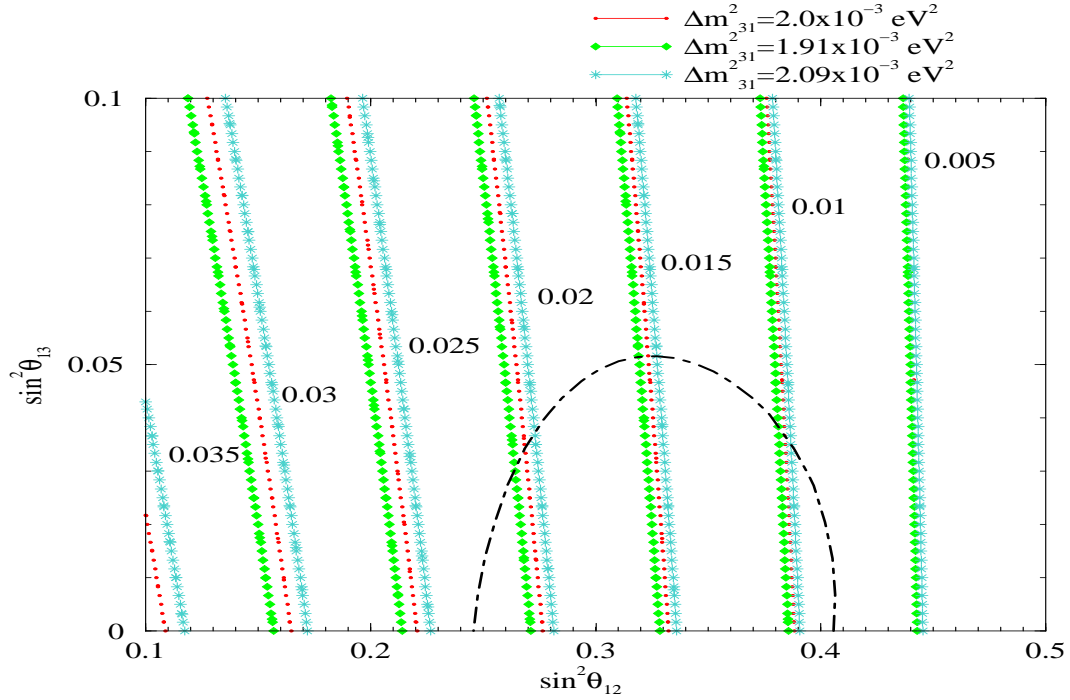

Figure 2: Lines of constant minimal value of $\langle m \rangle$ (in eV) in the IH case ($\langle m \rangle_{\mathrm{min}}^{\mathrm{IH}}$) for different Δm_{31}^2.
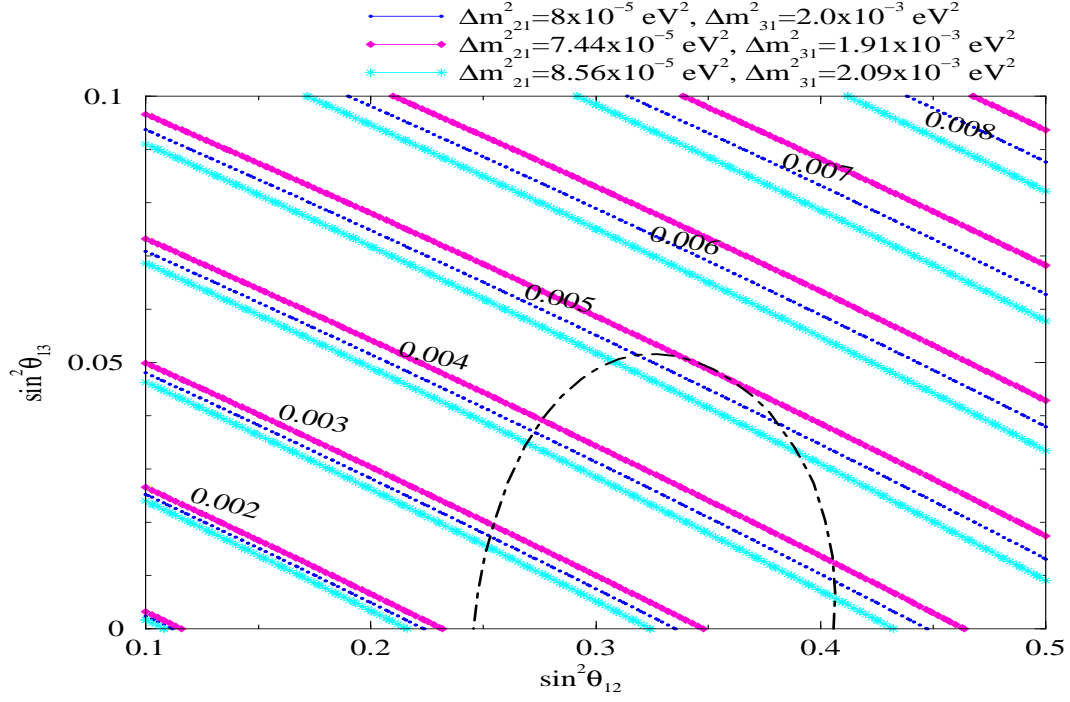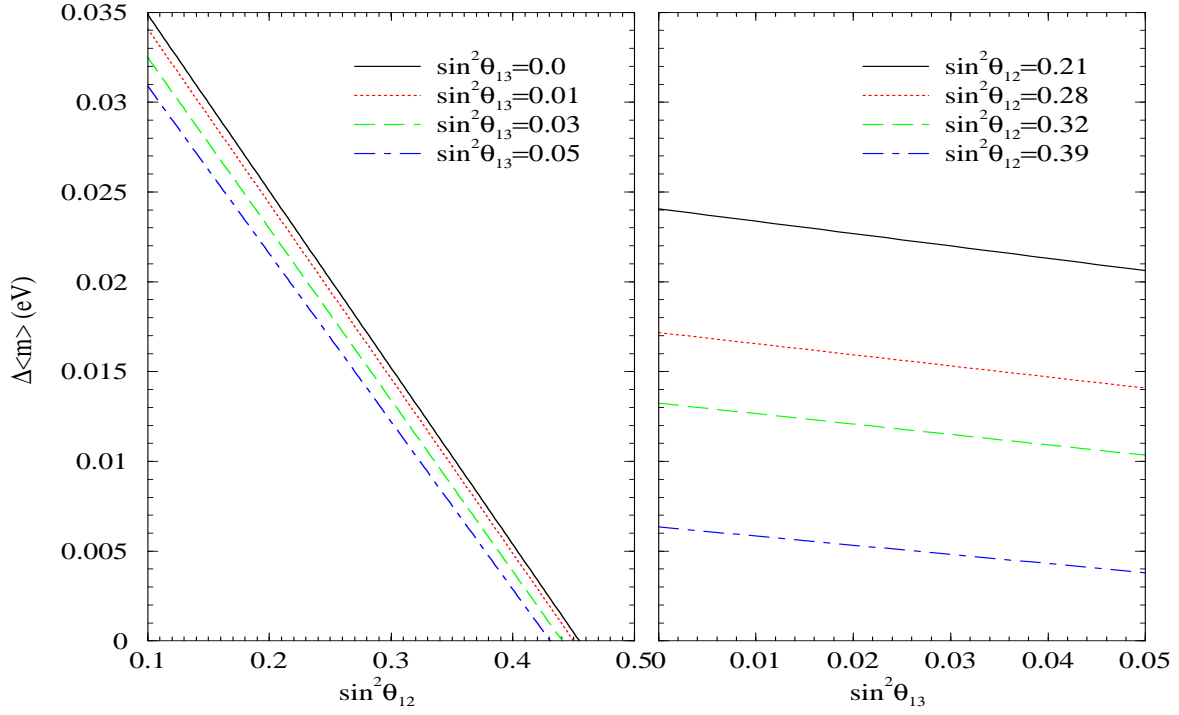
Figure 3: Lines of constant maximal value of $\langle m \rangle$ (in eV) in the NH case ($\langle m \rangle^{\mathrm{NH}}_{\max}$) for different Δm^2_{21}.
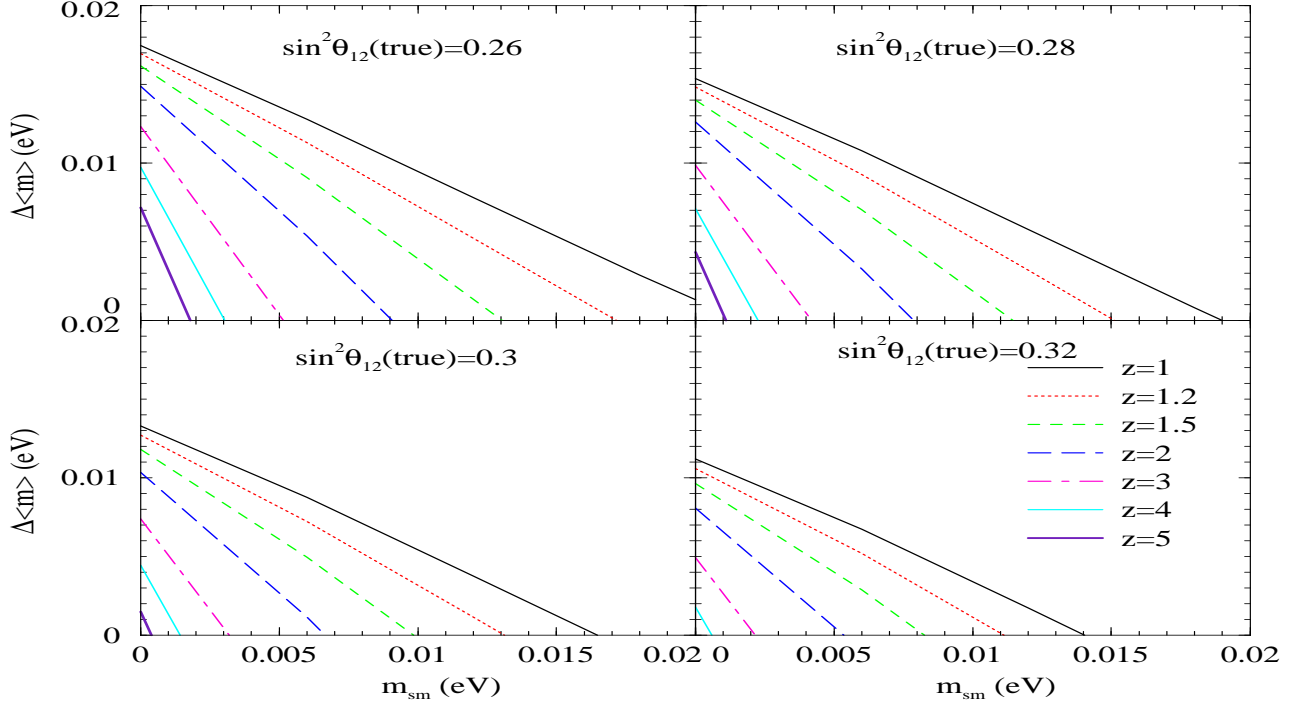

Figure 4: The difference between the minimal value of $\langle m \rangle$ for IH and the maximal value of $\langle m \rangle$ for NH, $\langle m \rangle^{\mathrm{IH}}_{\min} - \langle m \rangle^{\mathrm{NH}}_{\max}$, as a function of $\sin^2 \theta_{12}$ (left–hand panel) and $\sin^2 \theta_{13}$ (right–hand panel).
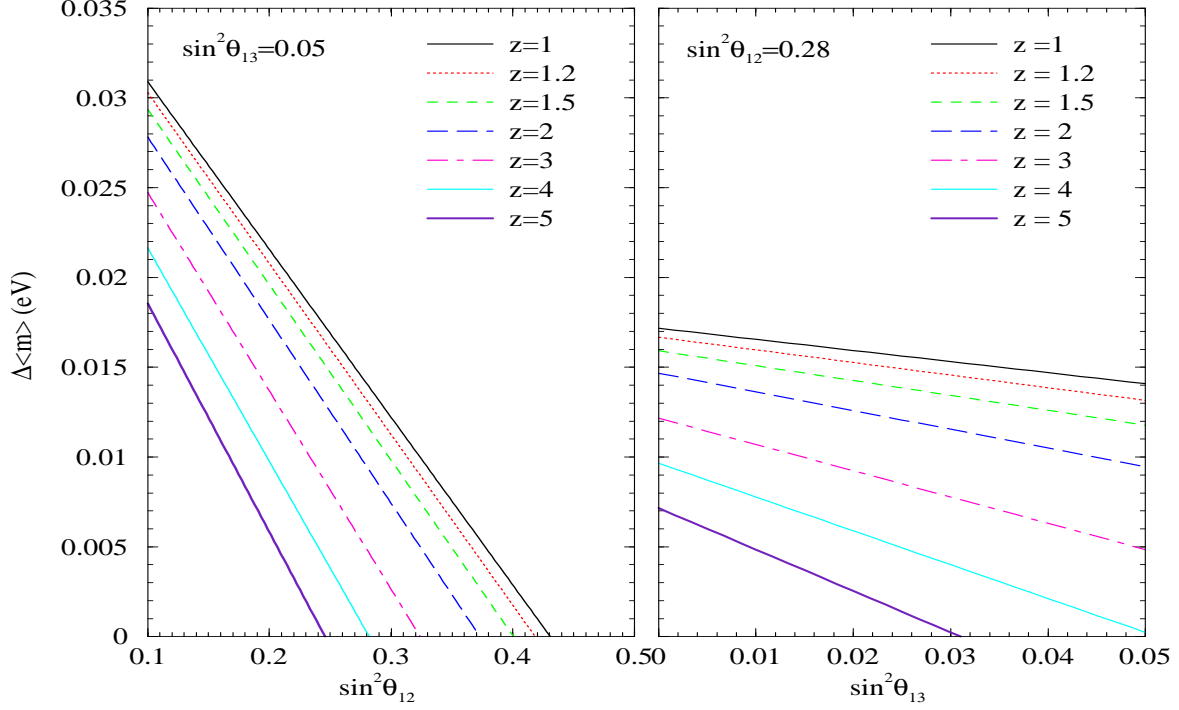
Figure 5: The difference between the minimal value of $\langle m \rangle$ for IH and the maximal value of $\langle m \rangle$ for NH times the NME uncertainty, $\langle m \rangle^{\mathrm{IH}}_{\mathrm{min}} - z \langle m \rangle^{\mathrm{NH}}_{\mathrm{max}}$, as a function of $\sin^2 \theta_{13}$ for different z and $\sin^2 \theta_{12} = 0.28$ (left–hand panel) and $\sin^2 \theta_{13}$ (right–hand panel).

Figure 6: The difference between the minimal value of $\langle m \rangle$ for IH and the maximal value of $\langle m \rangle$ for NH, $\langle m \rangle^{\mathrm{IH}}_{\mathrm{min}} - \langle m \rangle^{\mathrm{NH}}_{\mathrm{max}}$, as a function of the smallest neutrino mass m_{sm}. The 4 panels are for 4 assumed "true" values of $\sin^2 \theta_{12}$. The values of $\sin^2 \theta_{12}$ are allowed to vary within 6% of the assumed true value, while $\sin^2 \theta_{13}$ is allowed to vary between 0 and 0.0025.
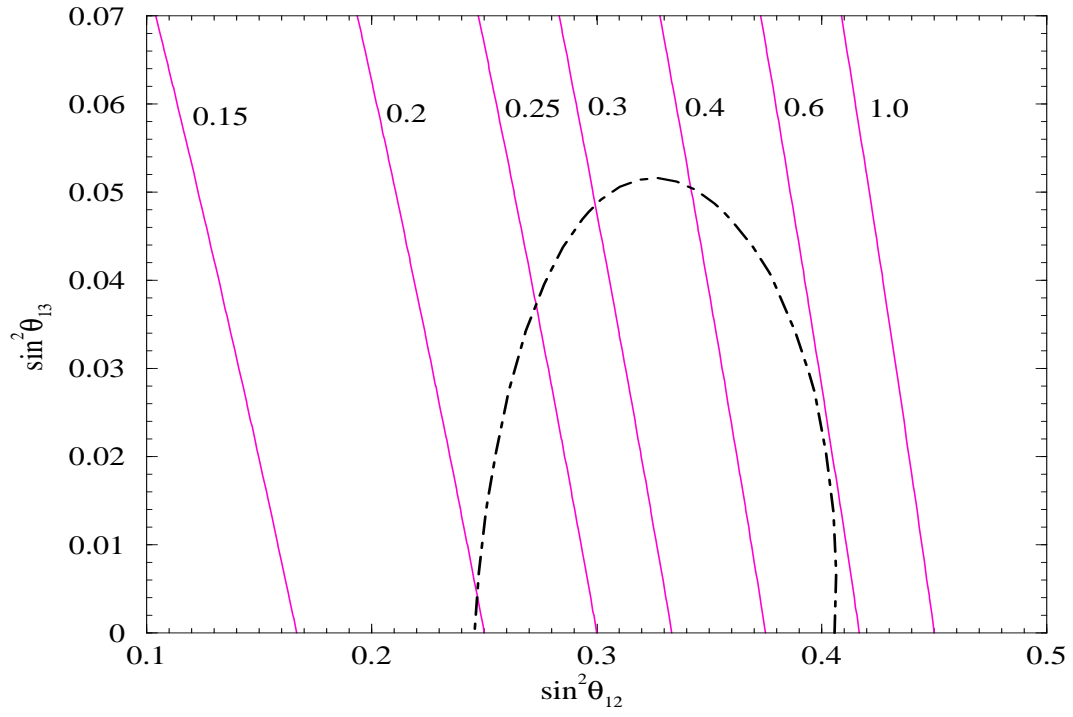
Figure 7: The lines of constant m_0 in the $\sin^2\theta_{12} - \sin^2\theta_{13}$ plane, predicted for a QD mass spectrum. Also shown is the current 3σ allowed region in $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$.
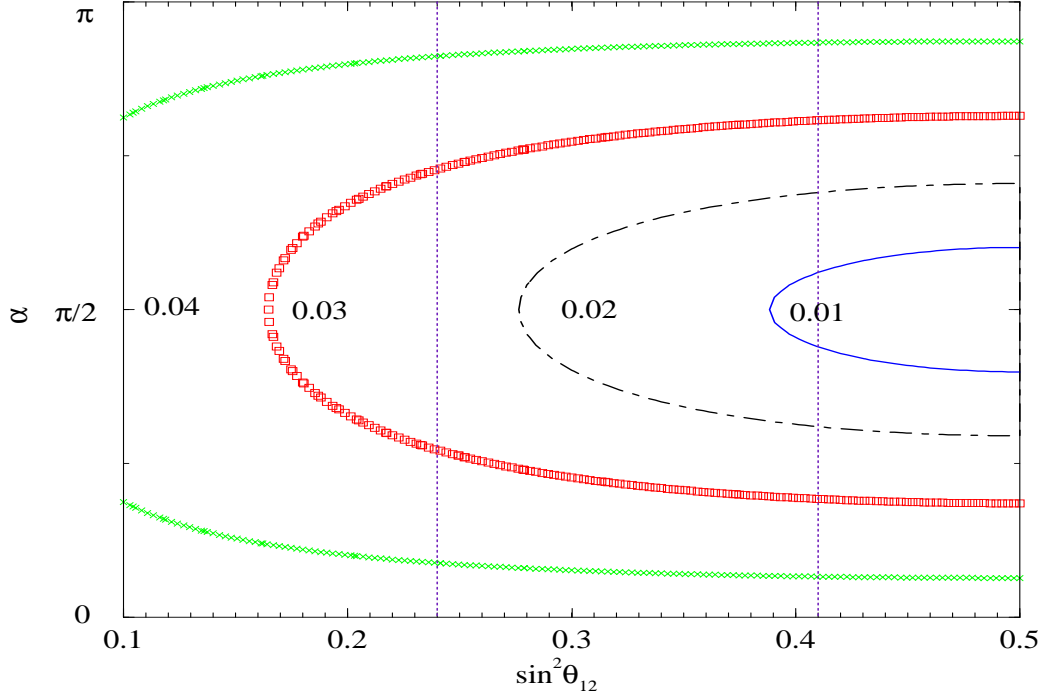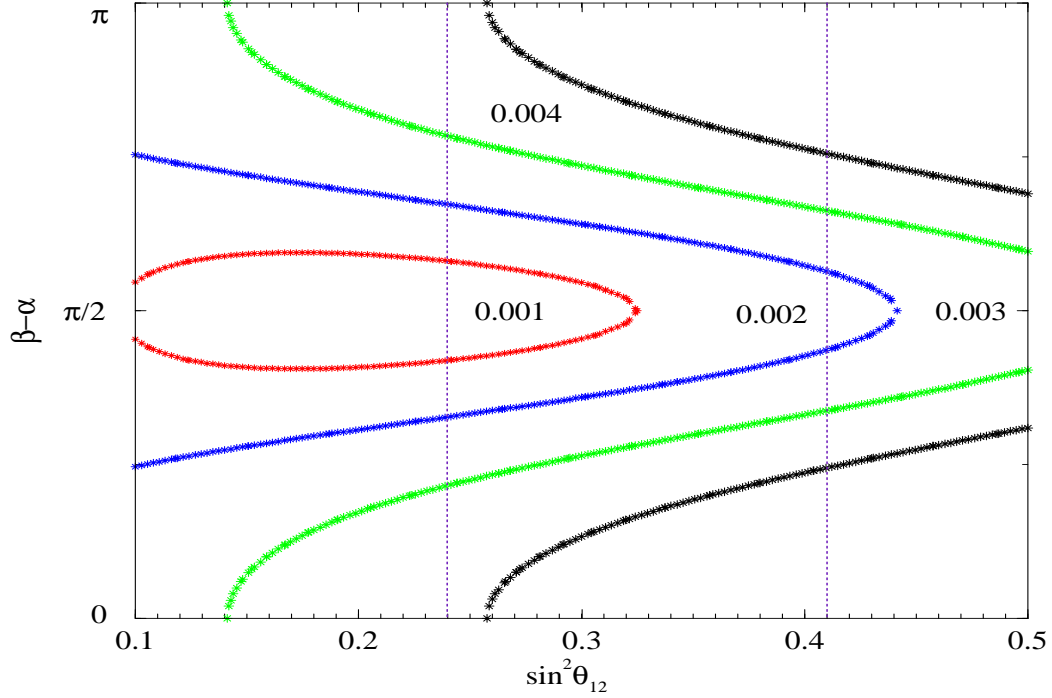
Figure 8: Allowed values of $\sin^2 \theta_{12}$ and the Majorana phase α for a small upper limit on $\langle m \rangle$ (in eV) in case of IH. The dotted lines show the current 3σ limit on $\sin^2 \theta_{12}$. The allowed areas are to the right of the respective curves.

Figure 9: Allowed values of $\sin^2 \theta_{12}$ and the Majorana phase combination $\beta - \alpha$ for a small upper limit on $\langle m \rangle$ (in eV) in case of NH. The dotted lines show the current 3σ limit on $\sin^2 \theta_{12}$. The allowed areas are to the left of the respective curves.
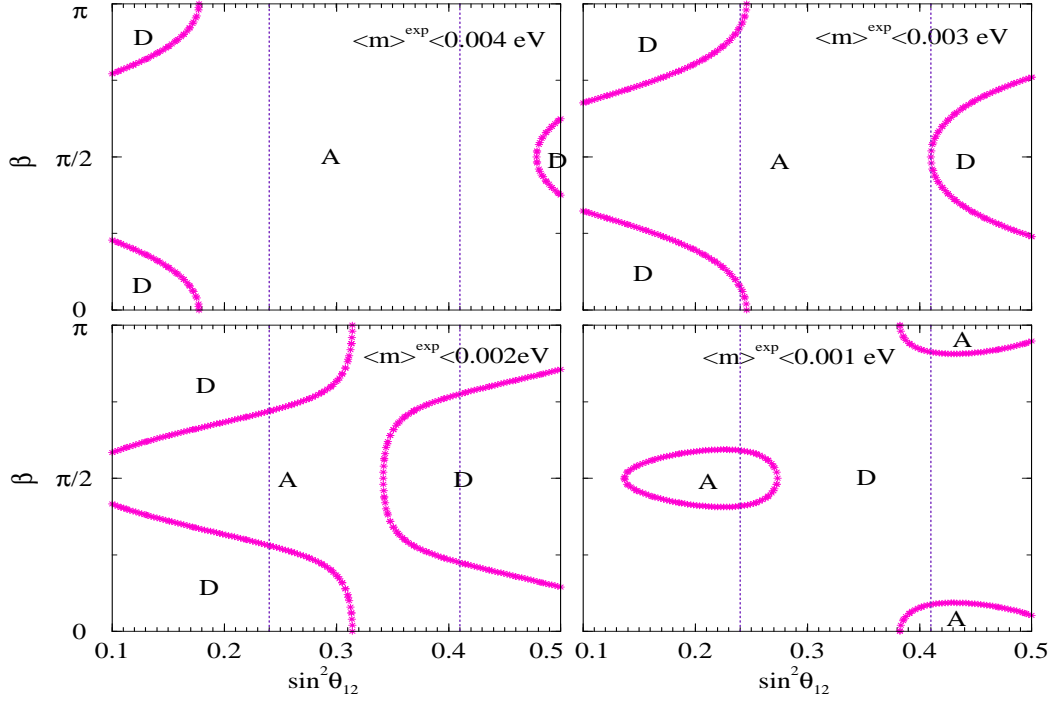
Figure 10: Allowed values of $\sin^2\theta_{12}$ and the Majorana phase β for a smallest neutrino mass of $m_1 = 0.005$ eV in case of NH for 4 different assumed upper limits on $\langle m \rangle$ and $\alpha = \pi/2$. The dotted lines show the current 3σ limit on $\sin^2\theta_{12}$. The allowed (disallowed) areas are marked with A (D).
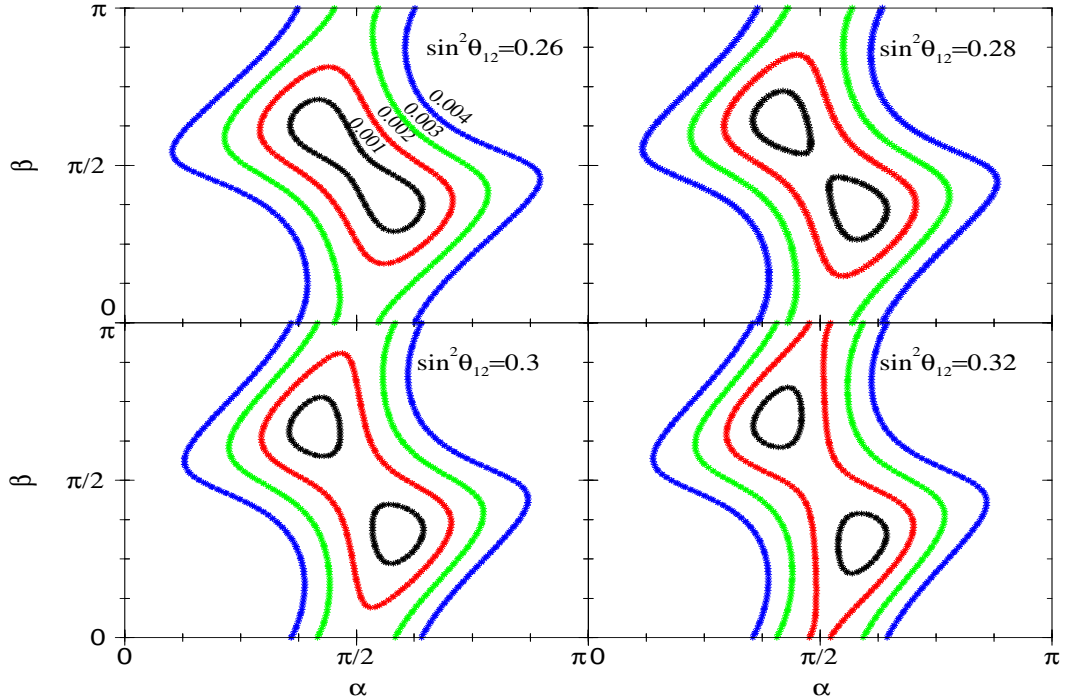

Figure 11: Allowed values of the Majorana phases α and β for a smallest neutrino mass of $m_1 = 0.005$ eV in case of NH for 4 different assumed upper limits on $\langle m \rangle$. The 4 panels correspond to 4 different true values of $\sin^2\theta_{12}$ shown on the Figure. Allowed are the areas within the respective curves.
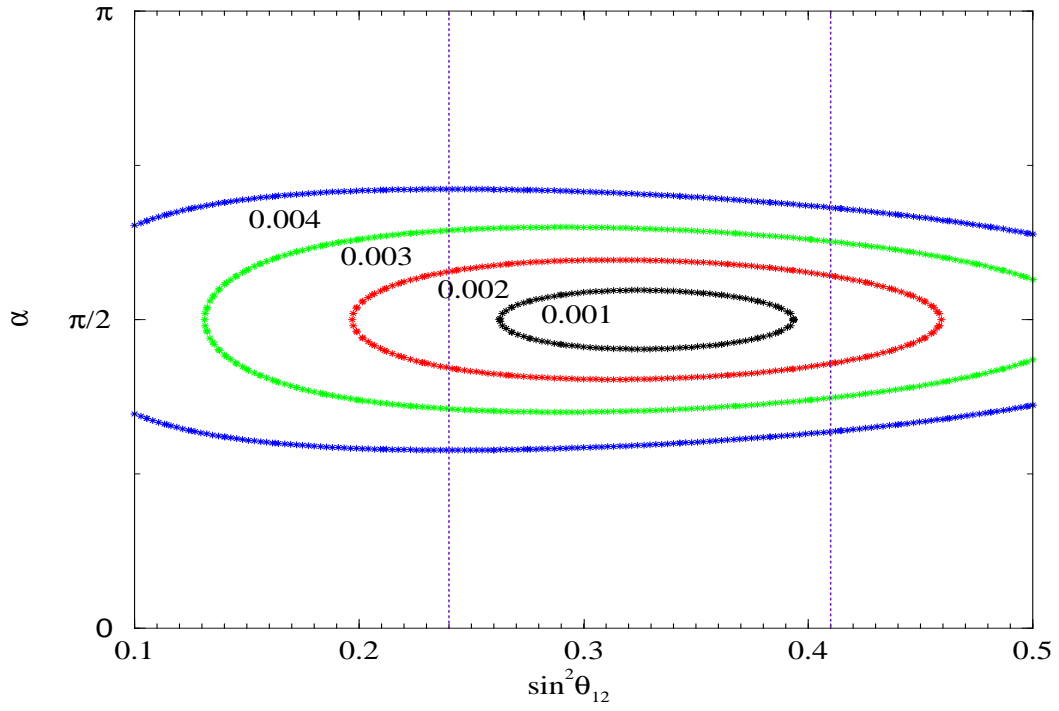
Figure 12: Allowed values of the Majorana phase α and $\sin^2 \theta_{12}$ for NH for $\sin^2 \theta_{13} = 0$ and with a non–zero smallest neutrino mass of $m_1 = 0.005$ eV in case of NH for 4 different assumed upper limits on $\langle m \rangle$. The dotted lines show the current 3σ limit on $\sin^2 \theta_{12}$. Allowed are the areas within the respective curves.